

MAGNETOHYDRODYNAMICS AND SOLAR PHYSICS

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Abstract. In this short review, I present some of the recent progresses on the pending questions of solar physics. These questions let us revisit the solar wind, the solar dynamo problem, the dynamics of the photosphere and finally have a glimpse at other solar type stars. Discussing the use of direct numerical simulations in solar physics, I show that the full numerical calculation of the flow in a single supergranule would require more electric power than the luminosity of the sun itself with present computer technology.

Keywords: solar physics, magnetohydrodynamics

1 Introduction

Studying the sun is motivated by many reasons. First, we would like to be able to explain to the street man, what is the sun, what has been its life until now, what will be its future, why it has permitted the appearance of life on Earth and whether it is unique or not in the Universe. These many reasons should be completed by the questions that stimulate astrophysicists in their quest of a full understanding of this celestial object. Indeed, the sun is also the closest star and it is a true self-operating physics laboratory where we can find conditions that cannot be reached on Earth.

Today the sun seems to be well-known: its fundamental parameters have been determined with some precision, not reached for any other star, and thanks to helioseismology, namely thanks to a careful interpretation of the frequencies of the tiny acoustic vibrations of the sun, we have also been able to check our calculations of its structure. It turns out that evolutionary models compare nicely to helioseismic models. Errors on basic thermodynamic quantities like temperature, density, pressure are around or less than 1% (Gough et al. 1996).

Of course the devil is in the details, and details are not missing on the sun. The first “big” detail is certainly its magnetic activity. If $\alpha - \Omega$ dynamo models allow us to retrieve the basic oscillation of the solar magnetic field, the understanding of irregularities of the cycle remains a challenge (Rieutord 2008). We understand that the cycle is strongly related to the differential rotation, but this feature of the dynamics of the sun still escapes a comprehensive view (although some numerical simulations can reproduce it – e.g. Brun & Toomre 2002). But among the challenges that the sun prompts to us, we should point out the origin of the supergranulation. This velocity feature has been known for more than fifty years (e.g. Rieutord & Rincon 2010), and we are still looking for the reasons of its existence. Last but not least, the problem of heating the sun’s corona is still a pending challenge.

These questions are actually important for human activities. It is indeed observed that the magnetic activity of the sun is related to the irradiance of the Earth (see figure 1) and it is believed that the rather cooler climate that happened in Europe in the period 1645-1715 is actually a consequence of the vanishing solar activity during that period (the so-called Maunder minimum e.g. Ribes & Nesme-Ribes 1993 or Beer et al. 1998). Of course all the present space activities are dependent on the particle flux emitted by the sun and should be protected against the coronal mass ejection. However, the magnetic field of the magnetically active sun is also a shield that prevents, in part, the galactic cosmic rays from reaching the Earth. This is an everyday life concern for aircraft pilots who face the gamma ray bath due to these cosmic rays (e.g. O’Brien et al. 1996).

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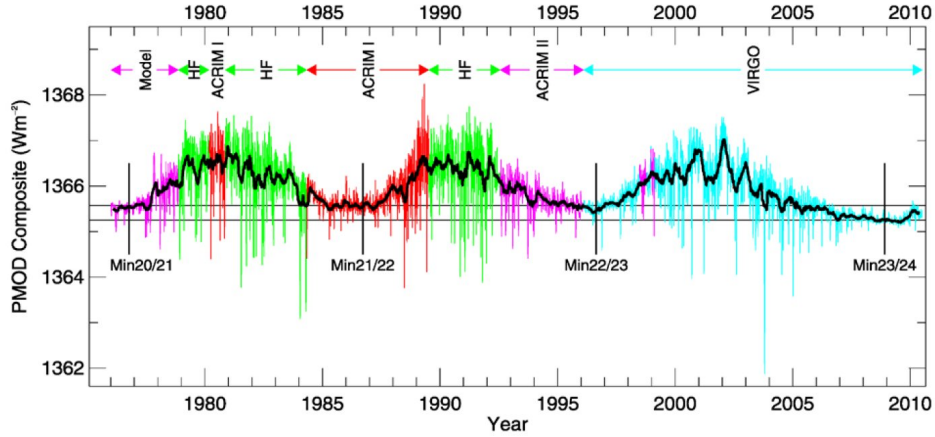


Fig. 1. Variation of the irradiance with time showing the imprint of the cycle (credit Fröhlich 2013).

2 The sun as a laboratory

The sun is a laboratory where we can observe matter in very extreme conditions compared to the terrestrial ones. In the old times, this allowed the discovery of helium by Janssen and Lockyer in 1868. More recently, the neutrinos oscillations have been discovered in the neutrinos emitted by the sun (e.g. Fukuda et al. 1998; Gough 2003). Yet, the sun is also a laboratory of giant size for studying turbulent fluid flows, or flows governed by magnetic fields, etc.

In particle physics, we are still looking for the theory that would unify, for instance, gravitation and the quantum world. In magnetohydrodynamics, the equations are well-known, namely,

$$\left\{ \begin{array}{l} \rho \frac{D\vec{v}}{Dt} = -\vec{\nabla}P + \mu\Delta\vec{v} + \vec{j} \wedge \vec{B} \\ \vec{\nabla} \cdot \vec{v} = 0 \\ \frac{\partial \vec{B}}{\partial t} = \text{Rot}(\vec{v} \wedge \vec{B}) - \text{Rot}(\eta \text{Rot} \vec{B}) \\ \vec{\nabla} \cdot \vec{B} = 0 \\ \rho \frac{De}{Dt} = \vec{\nabla} \cdot (\chi \vec{\nabla} T) - P \vec{\nabla} \cdot \vec{v} + \frac{\mu}{2} (\vec{\nabla} : \vec{v})^2 + \zeta (\vec{\nabla} \cdot \vec{v})^2 + \eta (\text{Rot} \vec{B})^2 / \mu_0 \end{array} \right. \quad (2.1)$$

but their general solutions are still a dream.

2.1 Solving for the fluid flows

Solar flows are characterized by very large Reynolds numbers* typically above 10^{10} . Let us consider in more details the challenge of computing the evolution of a single solar granule from the sole fluid mechanics equation. With typical size of 1000 km, a typical velocity of 1 km/s and a typical kinematic viscosity of $10^{-3} \text{ m}^2/\text{s}$ (e.g. Rieutord 2008), the Reynolds number is 10^{12} . The most energetic scale of the granule is of the size of the granule itself, namely 1000 km, and the scale at which viscosity smoothes velocity gradients is $\text{Re}^{-3/4}$ smaller, namely 1 mm. It is therefore clear that numerical simulations will never reach such a resolution, at least for two reasons. First, it is useless: we are not interested in such details, and it is likely that such details are unimportant. Second, it is energetically impossible with present computer technology as we shall see now.

To include the smallest vortices, we need a grid mesh about ten times smaller than the dissipative structure, thus of size equal to 0.1 mm. Kolmogorov scaling law predicts that velocity amplitude decreases with the one third power of the scale. Hence, from 1000 km to 1 mm the velocity fluctuations have been reduced by a

*We recall that the Reynolds number of a flow is the ratio VL/ν where V is a typical velocity scale of the flow, L is a typical length scale of the flow and ν is the kinematic viscosity of the fluid.

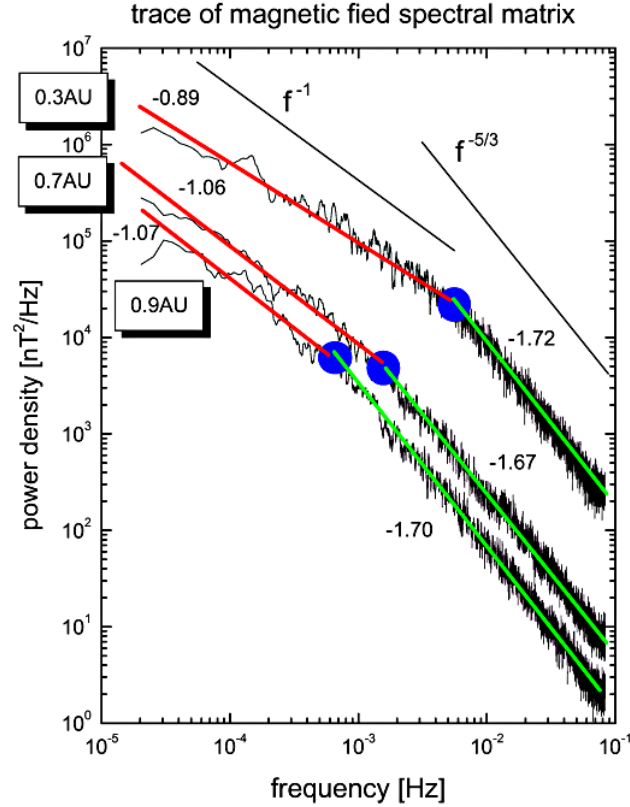


Fig. 2. Magnetic energy spectra as observed by Helios 2 in 1976 (from Bruno & Carbone 2005).

factor 10^3 , thus to 1 m/s. Taking care of these velocities needs a time step of 10^{-4} s according to the Courant-Friedrichs-Lewy criterion ($\delta t \leq \delta x/V$). To summarize, we need a box of size 1000 km with a grid mesh of 0.1 mm, that is 10^{30} grid points. The time step needs to be not larger than 10^{-4} s, so as to follow the flow in real time.

As for the code, we take the PENCIL code as an example (Brandenburg & Dobler 2002). This code needs, typically, 80 floating-point operations per time step per grid point. Thus, just to follow the sun on one of its granule, we would need a calculator with that produces 8×10^{35} flops, a number to be compared with the present most powerful machine that produces 4×10^{16} flops. The difference is enormous, but the problem is that of the needed energy to run 8×10^{35} flops with present technology. Such technology indeed can produce 75 Gflops per watt. Hence, the power needed would be of order 10^{25} watts = $0.025 L_{\odot}$ for a single granule! A single supergranule that contains a few hundred granules would need more than the power of the sun to be computed! Some colleagues mentioned to me the use of the quantum computer which may revolutionize the power needed for each flop, but it is not obvious that every algorithm will benefit from the efficiency of this computer.

The conclusion of this digression is that the modeling of the subgrid scales in turbulent flows remains a priority if we wish reasonable models of solar (and more generally of astrophysical) flows.

2.2 Three kinds of flows

As far as we know, turbulence modelling is not universal and therefore various and documented situations offer useful playgrounds to progress in our understanding of turbulent flows. As far as the sun is concerned, three regions may be observed and may lead to new guiding lines for turbulence modeling.

The first one may be the solar wind. This flow has been observed in situ by many space missions and celebrated spectra of the magnetic field fluctuations have been measured by Helios 2 (see Fig. 2). Such spectra are of interest because they guide us in the difficult problem of MHD turbulence. For instance, Iroshnikov (1964) and Kraichnan (1965) showed with phenomenological arguments, that the kinetic energy spectrum

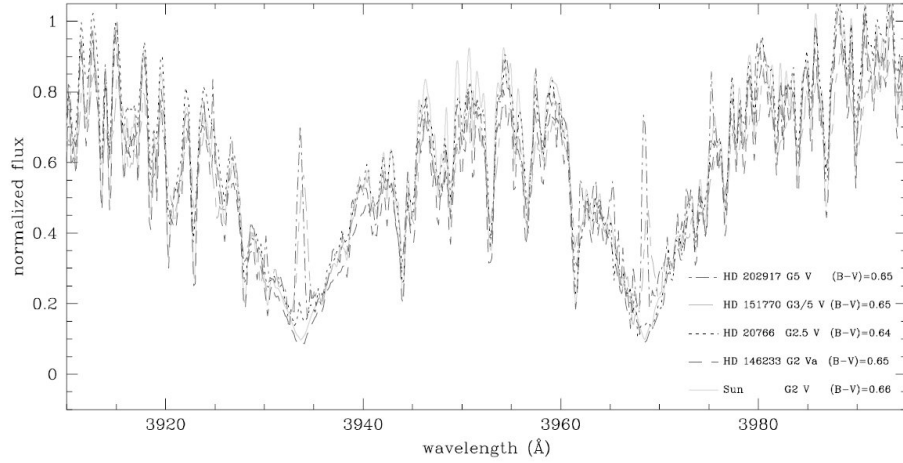


Fig. 3. The H and K absorption lines of Ca^+ for various solar-type stars (from Cincunegui & Mauas 2004). Note the thin emission line that arises in some stars right in the middle of the large absorption H and K lines.

should decrease like

$$E_k \propto k^{-3/2}$$

However this phenomenology was contested by Goldreich & Sridhar (1995) who showed that the anisotropy imposed by the magnetic field is crucial and therefore that $E_k \propto k_{\perp}^{-5/3}$, where k_{\perp} is the wave vector component orthogonal to the mean-field.

Grappin & Müller (2010) and Grappin et al. (2014) have studied this problem through turbulence modelling in the spectral space using an incompressible and perfect (non-diffusive) fluid. The point was to determine the role of the various parameters that intervene in this problem. Among other things, they show that the nature of the spectrum, and therefore its exponent, depends on the intensity of the background magnetic field and on the correlation time of the large-scale forcing. They could compute the spectra for various relative angles of the wave and magnetic vectors, showing the presence of a $k^{-3/2}$ scaling spreading over a decade (Grappin et al. 2014).

Different conditions may be found at the sun's surface, in the photosphere. There, the magnetic and velocity fields can both be measured and spectra obtained (e.g. Rieutord & Rincon 2010), but most detailed observations are for the velocity fields, thanks to granule tracking (e.g. Rieutord et al. 2007). There too, various characteristics of turbulent flows can be measured. For instance Rieutord et al. (2008) have determined the first spectrum of surface flows describing supergranulation, while Rieutord et al. (2010) have shown that the supergranulation peak disappears when a magnetic pore is in the field. In this same study the spectra of intensity fluctuations have also been derived, showing among other results that the exponent describing the subgranular scale depends on the wavelength used for the observation. On the theoretical side, the main success has certainly been the simulation of the solar photosphere so as to reproduce the line profiles of various elements and deduce new constraints on the solar abundances (Nordlund et al. 2009).

In between the solar wind and the solar photosphere are the chromosphere and the corona. In these regions numerous questions are raised by MHD phenomena. We cannot avoid mentioning the still pending heating of the solar corona for which Alfvén (or magnetoacoustic) waves are serious candidates for carrying the energy. The recent result of López Ariste et al. (2013) on the dislocations observed in propagating MHD waves may both enlight the heating of the corona and the question of the flux of magnetic helicity at the sun's surface. Indeed, such dislocations may carry some magnetic helicity and therefore contribute to the global flux of magnetic helicity at the surface of the sun. We recall that magnetic helicity, namely

$$H_m = \int_{(V)} \vec{A} \cdot \vec{B} dV$$

is an invariant of ideal MHD if the boundary of the fluid does not let any magnetic flux going through (i.e. if $\vec{B} \cdot \vec{n} = 0$ on the boundary). This is typically an (approximate) invariant of coronal loops. But this is a quantity

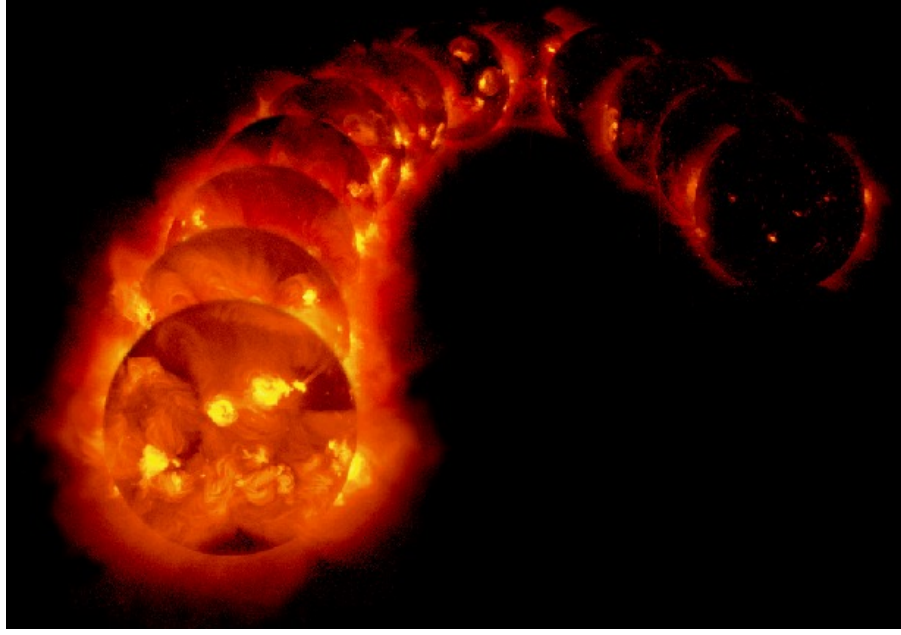


Fig. 4. The solar magnetic cycle as viewed in the X-rays by the satellite Yohkoh between 1991 and 2001.

that is important to measure so as to estimate its flux at the surface of the sun. Indeed, one of the recent results of numerical simulations of fluid dynamos is that saturation of the α -effect, the so-called α -quenching is affected by the magnetic helicity. If magnetic helicity cannot be expelled from the fluid domain, numerical simulations have shown that the α -effect is catastrophically quenched. At such a low level, this mechanism is no longer effective and would compromise the solar dynamo (Brandenburg & Subramanian 2005; Rieutord 2008). The sun manages to expell this helicity but the process is not well-known.

Hence, measuring the solar flux of magnetic helicity is crucial to put constraints on the solar dynamo. This is a difficult task that has been attempted by Dalmasse et al. (2014) for instance.

3 Moving to other stars

The sun itself shows only one example of a magnetically active star but astrophysicists would like a more general picture to appreciate, for instance, the effects of changing global parameters (mass, age, rotation, chemical composition, ...) on the magnetic activity.

A longstanding way of monitoring the magnetic activity of stars has been to measure the intensity of chromospheric lines, especially the H (396.85 nm) and K (393.368 nm) lines of the calcium ion Ca^+ (see figure 3). Understanding the magnetic activity of solar-like stars has become a crucial point for the detection of exo-planets because the magnetic activity raises the detection level of the radial velocity signature of a planet. As shown by Livingston et al. (2007), the emission inside the H&K line is quite nicely correlated to the solar cycle thus supporting the relevance of this index for monitoring the activity. Presumably, the emission line inside the large H & K absorption lines are coming from the chromosphere of the star but the process of this emission is not completely clear (Hall 2008).

Additional difficulties come from the modeling of the corona which is a region dominated by the magnetic fields. Global models of a corona like that of the sun are slowly emerging (Amari et al. 2013). These model are all the more welcome that the corona is the seat of the X-ray luminosity of solar-type stars. Such an emission is naturally another signature of the magnetic activity of stars. In X-rays the sun's luminosity is quite low, typically,

$$10^{-7}L_{\odot} \leq L_X \leq 10^{-6}L_{\odot}$$

but is varies with the cycle as nicely shown by the celebrated pictures obtained with the Yohkoh satellite (see figure 4). Since this X-ray emission is triggered by shock waves driven by flares in the corona of the stars,

simulation of unstable magnetic configurations are an appropriate tool to investigate the energy released by the associated flows (Pinto et al. 2014).

4 Conclusion

Back to the sun we may conclude that our star is indeed a gigantic laboratory for MHD. There is a huge quantity of available data, but it is quite scattered (Rieutord 2012). From these data, constraints on various high Reynolds number flows may be derived. This is a detailed view of an active low mass star which should lead to understanding how such an activity influences the star's environment and further constraints the habitability problem.

References

- Amari, T., Aly, J.-J., Canou, A., & Mikic, Z. 2013, *A&A*, 553, A43
- Beer, J., Tobias, S., & Weiss, N. 1998, *Solar Phys.*, 181, 237
- Brandenburg, A. & Dobler, W. 2002, *Computer Physics Communications*, 147, 471
- Brandenburg, A. & Subramanian, K. 2005, *Phys. Reports*, 417, 1
- Brun, A. S. & Toomre, J. 2002, *ApJ*, 570, 865
- Bruno, R. & Carbone, V. 2005, *Living Reviews in Solar Physics*, 2, 4
- Cincunegui, C. & Mauas, P. J. D. 2004, *A&A*, 414, 699
- Dalmasse, K., Pariat, E., Démoulin, P., & Aulanier, G. 2014, *Sol. Phys.*, 289, 107
- Fröhlich, C. 2013, *Space Science Rev.*, 176, 237
- Fukuda, Y., Hayakawa, T., Ichihara, E., et al. 1998, *Physical Review Letters*, 81, 1158
- Goldreich, P. & Sridhar, S. 1995, *ApJ*, 438, 763
- Gough, D. 2003, *Ast. Space Sc.*, 285, 341
- Gough, D. O., Kosovichev, A. G., Toomre, J., et al. 1996, *Science*, 272, 1296
- Grappin, R. & Müller, W.-C. 2010, *Phys. Rev. E*, 82, 026406
- Grappin, R., Müller, W.-C., Verdini, A., & Gürçan, Ö. 2014, *ArXiv e-prints* 1312.3459
- Hall, J. C. 2008, *Living Reviews in Solar Physics*, 5, 2
- Iroshnikov, P. S. 1964, *Sov. Astron.*, 7, 566
- Kraichnan, R. H. 1965, *Phys. Fluids*, 8, 1385
- Livingston, W., Wallace, L., White, O. R., & Giampapa, M. S. 2007, *ApJ*, 657, 1137
- López Ariste, A., Collados, M., & Khomenko, E. 2013, *Phys. Rev. Lett.*, 111, 081103
- Nordlund, Å., Stein, R. F., & Asplund, M. 2009, *Living Reviews in Solar Physics*, 6, 1
- O'Brien, K., Friedberg, W., Sauer, H., & Smart, D. 1996, *Environment International*, 22, S9
- Pinto, R. F., Vilmer, N., & Brun, A. S. 2014, *ArXiv e-prints* 1401.0916
- Ribes, J. C. & Nesme-Ribes, E. 1993, *A&A*, 276, 549
- Rieutord, M. 2008, *C. R. Physique*, 9, 757
- Rieutord, M. 2012, in *Astronomical Society of the Pacific Conference Series*, Vol. 461, *Astronomical Data Analysis Software and Systems XXI*, ed. P. Ballester, D. Egret, & N. P. F. Lorente, 457
- Rieutord, M., Meunier, N., Roudier, T., et al. 2008, *A&A*, 479, L17
- Rieutord, M. & Rincon, F. 2010, *Living Reviews in Solar Physics*, 7, 1
- Rieutord, M., Roudier, T., Rincon, F., et al. 2010, *A&A*, 512, A4
- Rieutord, M., Roudier, T., Roques, S., & Ducottet, C. 2007, *A&A*, 471, 687